1. (10 points) Calculate $\int \frac{4}{\sqrt{1 + e^x}} \, dx$

2. (20 points) Test this series for (a) absolute convergence (10 points) (b) conditional convergence (10 points)

$$\sum_{n=1}^{\infty} \frac{\cos (\pi n)}{n}$$

3. (10 points) Determine whether or not $f(x, y)$ has a limit at $(1,1)$

$$f(x, y) = \frac{x - y^4}{x^2 - y^4}$$

4. (20 points) Let $\Omega$ be the first-quadrant region bounded by the curves:

$$x^2 + y^2 = 4, \quad x^2 + y^2 = 16, \quad x^2 - y^2 = 1, \quad x^2 - y^2 = 4$$

Evaluate

$$\iint_{\Omega} \frac{1}{4} x^3 y \, dxdy$$

5. (20 points) Evaluate the line integral

$$\oint_C (1 + 20xy + y^2) \, dx + (5x^2) \, dy$$

where $C$ is the oriented counter clockwise square with vertices $(0,0), (1,0), (1,1), (0,1)$

6. (20 points) Calculate the flux of the vector field $\mathbf{v}(x, y, z) = xi + j + zk$ across the sphere $S: x^2 + y^2 + z^2 = 4$ in the outward direction.
1. Four particles move on a horizontal plane with the same constant acceleration $a\hat{y}$, where $a > 0$. At $t = 0$, their initial velocity vectors are $(a)12\hat{x} + 10\hat{y}$, $(b)12\hat{x} + 8\hat{y}$, $(c)8\hat{x} + 12\hat{y}$, $(d)12\hat{x} + 12\hat{y}$. If the respective displacements at $t > 0$ are $R_a, R_b, R_c, R_d$, which of the following is correct?
   (A) $R_a > R_b > R_c > R_d$, (B) $R_b > R_c > R_a > R_d$, (C) $R_d > R_a > R_c > R_b$, (D) $R_d > R_c > R_a > R_b$, (E) $R_a > R_b > R_c > R_d$.

2. Given the position vector $\vec{r} = (t^2 + 2)\hat{i} + (3t - 5)\hat{j} + (-t^2 - t + 1)\hat{k}$, what is the magnitude of acceleration at $t = 1$? (A) $\sqrt{5}$, (B) $\sqrt{14}$, (C) $2\sqrt{2}$, (D) $2$, (E) $0$.

3. What is the rotational inertia of a solid sphere of radius $R$ and mass $M$ about an axis passing through the center of the sphere? (A) $\frac{1}{2}MR^2$, (B) $\frac{2}{3}MR^2$, (C) $\frac{1}{4}MR^2$, (D) $\frac{2}{5}MR^2$, (E) $MR^2$.

4. A man of mass $m$, initially at rest, falls from a point at $(d, 0, 0)$. If the gravitational acceleration is $\vec{a}_g = -g\hat{z}$, find the angular momentum of the falling man about the origin at time $t$: (A) $dmg\hat{t}\hat{x}$, (B) $\frac{1}{2}dmg\hat{t}\hat{x}$, (C) $dmg\hat{t}\hat{y}$, (D) $\frac{1}{2}dmg\hat{t}\hat{y}$, (E) $-\frac{1}{2}dmg\hat{t}\hat{z}$.

5. Which wheel is at equilibrium? (A) $F \leftrightarrow 2F$, (B) $F \leftrightarrow 3F$, (C) $F \leftrightarrow 2F$, (D) $F \leftrightarrow 2F$, (E) $F \leftrightarrow 3F$.

6. Imagine that a tunnel is drilled from the north pole through the center of the Earth to reach the south pole, and a particle of mass $m$ is dropped from one end of the tunnel what is the period of oscillation?
   (A) $\frac{3\pi}{G\rho}$, (B) $\sqrt{\frac{3\pi}{4G\rho}}$, (C) $\sqrt{\frac{3}{4\pi G\rho}}$, (D) $\sqrt{\frac{3\pi}{2G\rho}}$, (E) $\sqrt{\frac{4\pi}{G\rho}}$.

7. A linear SHM takes place at a frequency of 2 Hz about $x=0$. At $t=0$, the displacement is $x(0)=3$ and the velocity is $v(0)=0$. What is $v(1)$? (A) 0, (B) $\pi$, (C) $2\pi$, (D) $3\pi$, (E) $6\pi$.

8. If the speed and frequency of a sound wave are $v$ and $f$ when measured by a detector at rest with respect to the source, what is the frequency measured by the detector moving at a speed of $v_d$ while the sound source is moving in parallel at $v_s$ with respect the rest frame? (A) $\frac{v + v_d}{v - v_s}f$, (B) $\frac{v + v_d}{v - v_s}f$,
   (C) $\frac{v + v_d}{v - v_s}f$, (D) $\frac{v - v_d}{v + v_s}f$, (E) $\frac{v + v_d}{v - v_s}f$.

9. Find the heat capacity of two metals, each of masses $M_1$ and $M_2$ and specific heat of $X_1$ and $X_2$, when they are connected: (A) $\frac{X_1}{M_1} + X_2$, (B) $\frac{X_1}{M_1} + \frac{M_2}{X_2}$, (C) $M_1X_1 + M_2X_2$, (D) $\frac{X_1 + X_2}{M_1 + M_2}$, (E) $\frac{X_1}{M_1 + M_2}$. 

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10. A particle of temperature \( T \) is located in an environment of temperature \( T_{env} \). What is the temperature dependence of thermal radiation energy that is emitted from the particle? (A) \( T^4 \), (B) \( T_{env}^4 \), (C) \( T^4 - T_{env}^4 \), (D) \( T \), (E) \( T_{env} \).

11. The speed of molecules in an ideal gas follows Maxwell's distribution law. Which of the following is correct when comparing the average speed \( v_a \), most probable speed \( v_p \), and root mean square speed \( v_{rms} \)? (A) \( v_p > v_a > v_{rms} \), (B) \( v_{rms} > v_a > v_p \), (C) \( v_{rms} > v_p > v_a \), (D) \( v_a > v_{rms} > v_p \), (E) \( v_p > v_{rms} > v_a \).

12. A point charge \( Q \) is located at a distance of \( R/2 \) from the center of a spherical shell of radius \( R \). If the shell is electrically neutral, what is the net flux of electric field through the sphere shell? (A) \( Q \), (B) \( e_0 Q \), (C) \( Q/e_0 \), (D) 0, (E) \( Q/2 \).

13. Three point charges held at three corners of an equilateral triangle with side \( d \). If \( Q_1 = +q \), \( Q_2 = +q \), \( Q_3 = -4q \), the total electrostatic energy is (A) \( \frac{6q^2}{4\pi e_0 d} \), (B) \( \frac{-6q^2}{4\pi e_0 d} \), (C) \( \frac{7q^2}{4\pi e_0 d} \), (D) \( \frac{-7q^2}{4\pi e_0 d} \), (E) 0.

14. An RC circuit consists of a resistor, a capacitor, and an ideal battery of \( emf \) \( e \) in series. What is the charge in the capacitor at time \( t = RC \) after it starts charging? (A) 0.37Ce, (B) 0.5Ce, (C) 0.63Ce, (D) Ce, (E) 0. (Note: \( e = 2.7183 \) and \( 1/e = 0.3679 \))

15. A long straight wire of radius \( R \) carries a uniformly distributed current \( i \). What is the magnetic field at a distance \( r < R \) from the center of the wire? (A) 0, (B) \( \frac{\mu_0 i}{2\pi r} \), (C) \( \frac{\mu_0 i}{2\pi r} \), (D) \( \frac{\mu_0 i r}{2\pi r^3} \), (E) \( \frac{\mu_0 i R}{2\pi r^3} \).

16. Which of what follows describes the Faraday's law? (A) \( \oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi}{dt} \), (B) \( \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 i \), (C) \( \oint \mathbf{B} \cdot d\mathbf{A} = 0 \), (D) \( \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 e_0 \frac{d\Phi}{dt} \), (E) \( \oint \mathbf{E} \cdot d\mathbf{A} = 0 \).

17. Consider a long solenoid of \( n \) turns per unit length carrying in it a current \( i \). What is the stored energy density of this solenoid? (A) 0, (B) \( \frac{1}{2} \mu_0 n^2 i^2 \), (C) \( \frac{1}{2} \mu_0 n i^2 \), (D) \( \mu_0 n^2 i^2 \), (E) \( \mu_0 n^2 i^2 \).

18. What is the angular frequency of resonance for a series RLC circuit? (A) \( \sqrt{RC} \), (B) \( \sqrt{LC} \), (C) \( \frac{1}{\sqrt{RC}} \), (D) \( \frac{1}{\sqrt{LC}} \), (E) \( \frac{R}{\sqrt{LC}} \).

19. An unpolarized light is incident from material 1 with refractive index \( n_1 \) into material 2 with index \( n_2 \). What is the angle of incidence with respect to the interface when the reflected light becomes fully polarized? (A) \( \cos^{-1} \frac{n_2}{n_1} \), (B) \( \sin^{-1} \frac{n_1}{n_2} \), (C) \( \cos^{-1} \frac{n_1}{n_2} \), (D) \( \tan^{-1} \frac{n_1}{n_2} \), (E) \( \tan^{-1} \frac{n_2}{n_1} \).

20. A single-slit, cut from a thin plate to a width \( a \), is illuminated by a plane wave of light with wavelength \( \lambda \) at normal incidence onto the plate. Find \( a \) if the first minimum appear at \( \theta \). (A) \( \frac{1}{2} \lambda \sin \theta \), (B) \( \lambda \sin \theta \), (C) \( \frac{\lambda}{2\sin \theta} \), (D) \( \frac{\lambda}{\sin \theta} \), (E) \( \frac{3\lambda}{2\sin \theta} \).